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Applying the Difference between the Convolutions of Test Signals and Object Responses to Investigate the Nonlinearity of the Transformation of Ultrawideband Signals

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Abstract—The nonlinearity of signal transformation by an object is investigated via sensing of the object by two successive test signals and determining the nonlinearity characteristic in terms of the difference between the convolutions of the test signals and the object responses. It is shown that, in such an approach, there is no need to prespecify the relationship between the test signals and predetermine the signal waveforms. An approach that takes into account the nonlinear signal distortions in a receiver is described. In an experimental investigation of the nonlinearity of the transformation of the test signal (magnetic field) by metal objects, it is shown that the considered nonlinearity characteristic is several times greater than the nonlinearity characteristic of signal transformation that can be obtained via determination of sideband spectral components.

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INTRODUCTION

The advantages of applying a multifrequency signal in nonlinear radar, which were revealed in [1], have spurred interest in the investigation of the potentialities of applying ultrawideband pulses as sensing signals in nonlinear radar. However, when an object simultaneously exhibits nonlinearity and reactance properties, it is necessary to find the actual cause of distortions in the signal waveform produced by the object: linear transformation or a combination of linear and nonlinear transformations. Implementation of a method [2] based on determining the difference between an object response and a test signal necessitates the fulfillment of the following condition: "the effective width of the test-signal spectrum must be inside the horizontal segment of the frequency response from the object under test." Otherwise, it is necessary to compensate for linear distortions of the test signal produced by the object. In practice, this compensation can be accomplished only for time-independent linear distortions with simple frequency dependence. To implement the methods for determining the statistical characteristics of the response from an object that is sensed by a random signal [3, 4], a large sensing duration is necessary. With the methods based on formation of local nulls in the test-signal spectrum [5–8], it is possible to observe only small portions of the results of nonlinear transformation that are concentrated in the neighborhoods of nulls.

In [9], a method of determining the nonlinearity characteristic of the transformation of ultrawideband signals with continuous spectra (including pulsed signals) is described. It is assumed that these signals experience linear distortions that have a complex frequency dependence and can vary in time. In this method, an object is sensed successively by two test signals and the difference between the object response to the first test signal and the linearly transformed response to the second test signal is determined. Practical application of the method is impeded owing to the complexity of test-signal generation. Indeed, the second test signal must be generated via a linear transformation of the first signal:

$$X_2(\omega) = K_1(\omega)X_1(\omega), \quad (1)$$

where $X_1(\omega)$ and $X_2(\omega)$ are the spectra of test signals and $K_1(\omega)$ is a specified function. However, the development of a test-signal generator capable of precisely accomplishing linear transformation (1) according to prescribed function $K_1(\omega)$ involves certain difficulties.

Furthermore, when $|K_1(\omega)|$ is limited, $X_2(\omega)$ will have all the nulls of $X_1(\omega)$. This condition restricts the possibilities for selecting test signals.

The goal of this paper is to consider the theoretical and experimental aspects of a modification of the method described in [9] that does not use a specified relationship between test signals and takes into account nonlinear signal distortions in the receiver.

1. THEORY

According to the method described in [9], an object is successively sensed by test signals $x_1(t)$ and $x_2(t)$. In the general case, a test-signal transformation performed by the object is expressed as

$$u(t) = S[x(t)],$$

where $x(t)$ is the test signal, $u(t)$ is the object response, and S is the operator that changes the time function of the test signal to the time function of the response. The receiver is assumed to have two (reference and measurement) channels that process, respectively, the test signals generated at the generator output and the object responses.

Nonlinearity characteristic $\varepsilon(t)$ of the signals transformation produced by the object is defined by the relationship

$$\varepsilon(t) = S_u[u_1(t)] * S_x[x_2(t)] - S_u[u_2(t)] * S_x[x_1(t)], \quad (2)$$

where S_u is the nonlinear operator of the measurement channel that changes the time function of the object response at the input of the receiver's measurement channel to the time function at the output of this channel; S_x is the nonlinear operator of the reference channel; $u_1(t)$ and $u_2(t)$ are the object responses to signals $x_1(t)$ and $x_2(t)$, respectively; and the asterisk designates convolution.

A. Linear Receiver

Let us first consider the properties of nonlinearity characteristic $\varepsilon(t)$ in the degenerate case, when

$$\begin{aligned} S_x[x_{1,2}(t)] &= h_x(t) * x_{1,2}(t), \\ S_u[u_{1,2}(t)] &= h_u(t) * u_{1,2}(t). \end{aligned} \quad (3)$$

Here, h_x and h_u are the impulse responses, respectively, in the reference and measurement channels of the receiver (in the absence of nonlinear signal distortions in the receiver channels). When an object linearly transforms signals and equalities (3) hold, $\varepsilon(t) \equiv 0$. Indeed, the object linearly transforms signals if

$$u(t) = h(t) * x(t), \quad (4)$$

where $h(t)$ is the impulse response of the object and the equality sign indicates the identity for $x(t)$. Substituting (3) into (2), we arrive at the formula

$$\begin{aligned} \varepsilon(t) = {} & h_u(t) * u_1(t) * h_x(t) * x_2(t) \\ & - h_u(t) * u_2(t) * h_x(t) * x_1(t). \end{aligned} \quad (5)$$

Substituting (4) into the right-hand side of (5), we obtain

$$\begin{aligned} & h_u(t) * u_1(t) * h_x(t) * x_2(t) \\ & - h_u(t) * u_2(t) * h_x(t) * x_1(t) \\ = {} & h_u(t) * h(t) * x_1(t) * h_x(t) * x_2(t) \\ & - h_u(t) * h(t) * x_2(t) * h_x(t) * x_1(t), \end{aligned} \quad (6)$$

$$\begin{aligned} & h_u(t) * h(t) * x_1(t) * h_x(t) * x_2(t) \\ & - h_u(t) * h(t) * x_2(t) * h_x(t) * x_1(t) \equiv 0. \end{aligned} \quad (7)$$

If $\varepsilon(t) \neq 0$ at least for some values of time t, signals transformation by the object is nonlinear because $\varepsilon(t) \neq 0$ leads to nonfulfillment of (6) (obviously, (5) and (7) hold as before). Hence, substitution

$$u_1(t) = h(t) * x_1(t), \quad (8)$$

$$u_2(t) = h(t) * x_2(t) \quad (9)$$

is incorrect; i.e., (8) and (9) are jointly unsatisfiable. Therefore, (4) is not identical for $x(t)$. Hence, in this case, signals transformation is nonlinear.

However, the converse statement is not true; i.e., a nonlinear transformation of signals $x_1(t)$ and $x_2(t)$ does not guarantee that $\varepsilon(t) \neq 0$. For example, nonlinearity characteristic $\varepsilon(t)$ is equal to zero when these signals are transformed nonlinearly but $x_1(t) = x_2(t)$. Hence, signals $x_1(t)$ and $x_2(t)$ with different waveforms and/or amplitudes must be selected to obtain a nonzero value of $\varepsilon(t)$ at nonlinear signals transformation. This condition ensures different nonlinear transformations of these signals.

With allowance for the foregoing, the described approach is implemented as follows. If it is found that $\varepsilon(t) \neq 0$, signal transformation is inferred to be nonlinear.

In this case, fulfillment of relationship (1) is not required, thereby simplifying generation of test signals. Furthermore, there is no need to use test signals with prescribed waveforms. (In particular, nonlinear signal distortions in the generator are acceptable.) This circumstance enables us to investigate, for example, the nonlinearity of signal transformation in communications systems using the fragments of real signals transmitted in these systems (including signals with nonoverlapping spectra) as $x_1(t)$ and $x_2(t)$. Test signals can be realizations of a random process.

B. Nonlinear Receiver

Let us first consider a linear transformation of signals by an object (i.e., formulas (8) and (9) hold jointly) in the case when $\varepsilon(t) \neq 0$. It follows from the joint fulfillment of (8) and (9) that (6) and (7) hold as well. Hence, inequality $\varepsilon(t) \neq 0$ suggests the nonfulfillment of (5) obtained via substitution of (3) into (2). Since formula (2) is the definition of $\varepsilon(t)$, at least one of equalities (3) does not hold; i.e., signals are distorted nonlinearly at least in one receiving channel. When S is linear (i.e., when $S[x(t)] = h(t) * x(t)$, $\varepsilon(t)$ characterizes only the nonlinearity of signal transformation by the receiver. In this case, $\varepsilon(t)$ is designated as $\varepsilon_0(t)$:

$$\varepsilon_0(t) = S_u[h(t) * x_1(t)] * S_x[x_2(t)] - S_u[h(t) * x_2(t)] * S_x[x_1(t)]. \quad (10)$$

Let us investigate the nonlinearity of signal transformation by an object in the presence of nonlinear signal distortions in the receiver channels. We replace the object under study with an undoubtedly linear object and select impulse response $h(t)$ of this object so as to obtain variations in the waveform and amplitude of one test signal that are similar to those produced by the object under study:

$$h(t) * x_1(t) = u_1(t) \text{ or } h(t) * x_2(t) = u_2(t), \quad (11)$$

where $u_{1,2}$ are the responses from the object under study. If, at some values of time t, $\varepsilon(t)$ obtained for the object under study differs from $\varepsilon_0(t)$ corresponding to $h(t)$ that satisfies (11), signals are nonlinearly transformed by the object under study. Indeed, having substituted formulas (2) and (10) into the inequality $\varepsilon(t) \neq \varepsilon_0(t)$, we obtain

$$S_u[u_1(t)] * S_x[x_2(t)] - S_u[u_2(t)] * S_x[x_1(t)]$$

$$\neq S_u[h(t) * x_1(t)] * S_x[x_2(t)] - S_u[h(t) * x_2(t)] * S_x[x_1(t)].$$

It follows from the above inequality that equalities (11) do not hold jointly. Therefore, the equation $h(t) * x(t) = u(t)$ is not identical for $x(t)$. Thus, the signal transformation by the object under study is nonlinear.

In practice, it is not easy to choose a linear object with impulse response $h(t)$ exactly satisfying (11). Hence, it is suitable to find $\varepsilon_0(t)$ for $h(t)$ satisfying either the condition $h(t) * x_1(t) \approx u_1(t)$ or the condition $h(t) * x_2(t) \approx u_2(t)$. It is allowable in the case that characteristic $\varepsilon_0(t)$ varies insignificantly if the waveform and amplitude of $h(t) * x_1(t)$ or $h(t) * x_2(t)$ differ from those of $u_1(t)$ or $u_2(t)$ (i.e., these variations must not affect the decision about similarity or dissimilarity of $\varepsilon(t)$ and $\varepsilon_0(t)$). Here, as in (11), $u_{1,2}$ are the responses from the object under study.

C. Comparison between ε(t) and the Nonlinearity Characteristics Obtained via Determination of Sideband and Harmonic Components

Let us compare $\varepsilon(t)$ and the nonlinearity characteristics of signal transformation produced by an object that are obtained via determination of the sideband and harmonic components of the object response. This comparison can be implemented if, at the frequencies under study, $|F\{S_x[x_2(t)]\}| > 0$, where F is the Fourier transform. Let us convolve $\varepsilon(t)$ with the function

$$F^{-1}\left[\frac{1}{F\{S_x[x_2(t)]\}}\right],$$

where F^{-1} is the inverse Fourier transform. The result of this convolution, $\varepsilon^*(t)$, is expressed as

$$\varepsilon^*(t) = \varepsilon(t) * F^{-1}\left[\frac{1}{F\{S_x[x_2(t)]\}}\right].$$

Substituting (2) into the above formula and simplifying it, we obtain

$$\varepsilon^*(t) = S_u[u_1(t)] - F^{-1}\left[\frac{F\{S_u[u_2(t)]\}}{F\{S_x[x_2(t)]\}}\right] * S_x[x_1(t)]. \quad (12)$$

For object response $u(t)$ to multifrequency test signal $x(t)$, the sum of sideband and harmonic components can be interpreted as the residual of Eq. (4): $u(t) - h(t) * x(t)$, where $h(t)$ is the impulse response of an object satisfying the equality $F[h(t)] = U(\omega)/X(\omega)$ at all values of ω for which $|X(\omega)| > 0$. Here, $X(\omega)$ and $U(\omega)$ are the spectra of a multifrequency test signal and the object response.[1] Nonlinearity characteristic (12) can be interpreted as the residual of the equation

$$S_u[u(t)] = F^{-1}\left[\frac{F\{S_u[u_2(t)]\}}{F\{S_x[x_2(t)]\}}\right] * S_x[x(t)], \quad (13)$$

which is derived at $u(t)$ and $x(t)$ for $u_1(t)$ and $x_1(t)$. Equations (13) and (4) are identical if, in (4), test signal $x(t)$, $h(t)$, and object response $u(t)$ correspond to test signal processed by the receiver $S_x[x(t)]$, $F^{-1}\left[\frac{F\{S_u[u_2(t)]\}}{F\{S_x[x_2(t)]\}}\right]$, and object response processed by the receiver $S_u[u(t)]$, respectively. From this viewpoint, nonlinearity characteristic (12) and the nonlinearity characteristics obtained via determination of sideband and harmonic components differ only in the test signals used for sensing and in the rules used for selecting the linear approx-

[1] For this definition of function $h(t)$ and under the assumption that $h(t)$ has a limited amplitude spectrum, the spectrum of function $h(t) * x(t)$ is identical to that of object response $u(t)$ at the frequencies of multifrequency test signal $x(t)$ and equal to zero at other frequencies. Hence, the residual $u(t) - h(t) * x(t)$ is the sum of the harmonics and sideband components of the object response whose frequencies differ from frequencies of the multifrequency test signal.

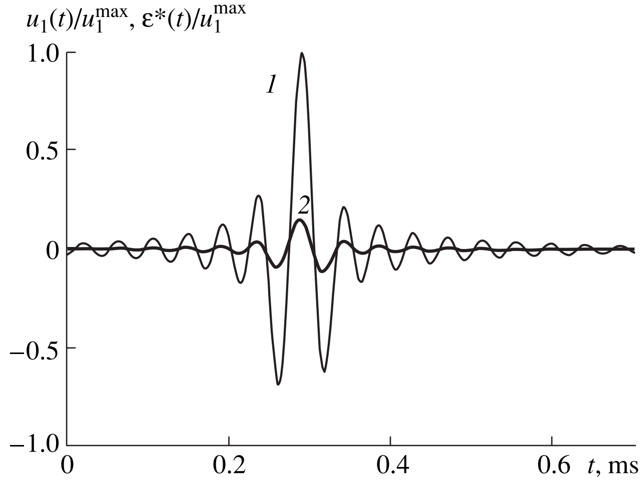

Fig. 1. Normalized (curve *1*) response $u_1(t)$ and (curve *2*) nonlinearity characteristic $\varepsilon^*(t)$ of a low-carbon-steel object.

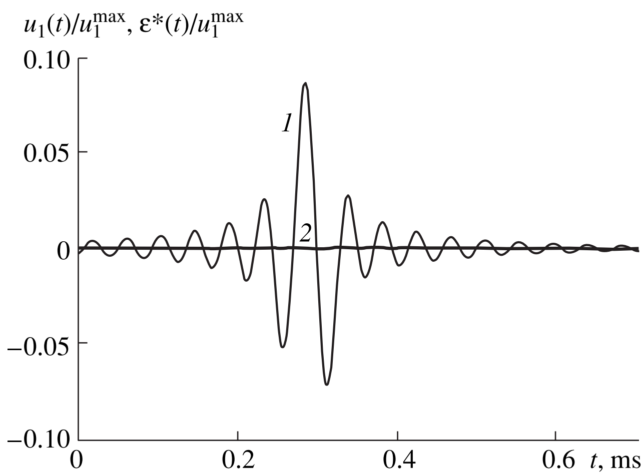

Fig. 2. Normalized (curve *1*) response $u_1(t)$ and (curve *2*) nonlinearity characteristic $\varepsilon^*(t)$ of an aluminum object.

imation of nonlinear transformation. This circumstance enables us to compare these characteristics.

2. EXPERIMENTAL

The adaptability of the proposed nonlinearity characteristic for revealing the additional classification features of metal objects was investigated experimentally. This characteristic was compared with the nonlinearity characteristic obtained via determination of sideband components in the object response to a multifrequency test signal. (It has been demonstrated [1] that, in nonlinear radar, a multifrequency signal offers advantages over a single-frequency signal.)

The experimental setup involved an generator, a receiver, and transmitting (57 turns) and receiving (188 turns) flat inductor coils of diameter 10 mm each. The coils were mounted in immediate proximity to each other (contacted each other), and their end surfaces were in the same plane. The oscillator's output signal was fed to the transmitting coil and to the input of the receiver's reference channel. In the operating frequency range, the transfer function of the transmitting coil–receiving coil system could be satisfactorily approximated by the function $H(j\omega) = H_0 j\omega\tau/(1 + j\omega\tau)$ with $\tau = 7.7$ μs and $H_0 = 0.22$. To compensate for the signal induced in the receiving coil by the transmitting coil, the difference between the generator signal passed through a passive filter with nonlinear transfer function $H(j\omega)$ and the output signal of the receiving coil was fed to the input of the receiver's measuring channel. To determine the nonlinearity characteristics of signals transformation produced by the receiver, compensation for an induced signal was temporarily violated by means of changing the filter transfer function so that the amplitude and waveform of the input signal of the receiver's measurement channel approximately corresponded to those of the object response.

Test signal $x_1(t)$ was used in the form

$$x_1(t) = \frac{\sin(2\pi f_{up} t)}{2\pi f_{up} t} - \frac{\sin(2\pi f_{up} t - \pi)}{2\pi f_{up} t - \pi},$$

where $f_{up} = 24$ kHz is the upper frequency limit of the spectrum of signal $x_1(t)$. The amplitude spectrum of test signal $x_2(t)$ was analogous to the amplitude spectrum of signal $x_1(t)$, and the phase spectrum of the former signal differed from the phase spectrum of $x_1(t)$ by a value that had a quadratic frequency dependence:

$$X_2(\omega) = X_1(\omega)\exp(-jd_2\omega|\omega|),$$

where d_2 is the coefficient [10] that determines a decrease in the amplitude of signal $x_2(t)$ and an increase in the duration of this signal relative to the corresponding parameters of signal $x_1(t)$. Coefficient d_2 was chosen to be 2.04×10^{-6} s^2. The amplitude of signal $x_2(t)$ was 1.7% of the amplitude of signal $x_1(t)$. Thus, signals $x_1(t)$ and $x_2(t)$ had substantially different amplitudes and waveforms. The maximum voltage of pulse $x_1(t)$ applied to the transmitting coil with a resistance of 6.3 Ω was 28 V.

To compare the proposed nonlinearity characteristic and the nonlinearity characteristic that was obtained via determination of sideband components, a two-frequency (16 and 18 kHz) test signal was used. Its amplitude was equal to the amplitude of signal $x_1(t)$. The necessary frequency resolution was achieved through selection of the duration of the two-frequency signal such that its value was much greater than the duration of signal $x_1(t)$. At a level of 0.1 of the amplitude of the two-frequency signal, its duration was 3.9 ms. Accordingly, the energy of the two-frequency signal was greater than the energy of signal $x_1(t)$.

Experimental results

Research subject	Normalized amplitude, %		
	response $u_1(t)$	nonlinearity characteristic $\varepsilon^*(t)$	sum of sideband components
Low-carbon-steel object	100	15.8	2.2
Testing of the inherent nonlinearity of the experimental setup performed with a signal resembling the response from the low-carbon-steel object	101	0.09	0.04
Aluminum object	8.7	0.48	0.25
Testing of the inherent nonlinearity of the experimental setup performed with a signal resembling the response from the aluminum object	9.4	0.46	0.23

As the object models, 10-mm-dia, 1-mm-thick low-carbon-steel and aluminum disks were used. The surface of the aluminum disk was covered with an oxide film. The object was placed above the coils at a distance of 2.5 mm from their end surfaces. For the low-carbon-steel and aluminum objects, responses $u_1(t)$ to test signal $x_1(t)$ and nonlinearity characteristic $\varepsilon^*(t)$ are shown in Figs. 1 and 2, where the responses of the objects and nonlinearity characteristics are normalized to amplitude u_1^{max} of response $u_1(t)$ of the low-carbon-steel object. The amplitudes of the object responses and nonlinearity characteristics shown in Figs. 1 and 2 are presented in the table, where the amplitudes of the object responses are likewise normalized to the amplitude of response $u_1(t)$ of the low-carbon-steel object, while the amplitude of the nonlinearity characteristic of each object is normalized to the amplitude of response $u_1(t)$ from the corresponding object. In addition, the table contains the amplitudes of the sum of sideband components in the response from each object to the two-frequency test signal. (These amplitudes are normalized with respect to the amplitude of the response from the corresponding object to the two-frequency test signal.) The inherent nonlinearity of the experimental setup was tested. The normalized results of testing (normalization was performed by analogy with the normalization of the responses and nonlinearity characteristics of the objects) are presented as well in the table.

CONCLUSIONS

Difference $\varepsilon(t)$ between the convolutions of test signals with object responses to these signals characterizes the nonlinearity of ultrawideband signal transformation produced by an object. (Here, the case of the presence of linear signal distortions performed by the object is included.) When $\varepsilon(t)$ is chosen as the nonlinearity characteristic of signal transformation, there is no need to use test signals with predetermined waveforms. (In particular, nonlinear signal distortions in an generator are acceptable.)

In the presence of nonlinear distortions in the receiver channels, the attribute of the nonlinearity of signal transformation produced by an object is the difference between $\varepsilon(t)$ obtained for the object under study and $\varepsilon(t)$ for an undoubtedly linear object that changes the waveform and amplitude of either of test signals in the same way as the object under study.

In the experiments performed with the help of the method described above, a significant nonlinearity of signals transformation by a low-carbon-steel object was revealed, while attributes of the nonlinearity of signal transformation performed by an aluminum object were not found. Hence, the proposed nonlinearity characteristic of signals transformation can be used to obtain additional classification attributes of an object.

When the low-carbon-steel object was sensed by a two-frequency test signal with an amplitude equal to the amplitude of $x_1(t)$, the normalized amplitude of the sum of sideband components in the object response was 2.2%. This value is 7 times less than the normalized amplitude of nonlinearity characteristic $\varepsilon^*(t)$ that was obtained for this object, although both the sum of sideband components and $\varepsilon^*(t)$ can be interpreted as the residuals of the linear equation used to approximate nonlinear transformation. One of the possible causes of this difference is that, in contrast to the described method, the nonlinearity characteristic representing a sum of sideband components does not contain spectral components of the object response with frequencies corresponding to those of the test signal. In conclusion, the described method has a practical advantage over the method in which the nonlinearity of signal transformation is investigated via determination of sideband components.

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